

$O(1/N^3)$ conformal bootstrap solution of the
 $SU(2) \times SU(2)$ Nambu–Jona-Lasinio model.

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Abstract. Using the full conformal bootstrap method an analytic expression is given in d -dimensions for the anomalous dimension of the fermion at $O(1/N^3)$ in a large N expansion of the Nambu–Jona-Lasinio model with $SU(2) \times SU(2)$ continuous chiral symmetry.

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Four fermi interactions have become important recently in various areas of field theory and its applications to particle and condensed matter physics. For example, in the former area it had been proposed that the Higgs mechanism of the standard model and the consequent mass generation could be reproduced, [1, 2, 5], by a top quark condensate which had its origins in an interaction involving four top fields. Originally such models were introduced in a different context by Nambu and Jona-Lasinio, [6], as a low energy effective theory describing hadronic physics, [7]. Although these considerations are four dimensional, four fermi theories also in fact play a role in understanding lower dimensional phenomena. For example, one interest in the three dimensional model centres on trying to ascertain the effect such an interaction induces in models describing high T_c superconductivity as it is believed to figure in that mechanism, [8]. In another direction, numerical simulations have been performed on the lattice for models with small numbers of flavours to observe the onset of non-perturbative effects and critical exponents have also been measured, [9]. Analytic calculations using several orders of the large N expansion are in fairly good agreement with such results, [11]. Further, the four fermi model, the Gross Neveu model, [10], is in the same universality class as the infra-red fixed point of the Yukawa model in all dimensions d , $2 < d < 4$, [3, 4, 5]. This equivalence is verified through ϵ -expansion techniques and knowledge of the perturbative structure which is also used to gain improved estimates of critical exponents. Indeed there has been recent work in this direction, [11-22], where the ϵ -expansion of the $O(N)$ Gross Neveu model and the related Nambu–Jona-Lasinio models with various continuous chiral symmetries have been examined from the point of view of establishing equivalences with various discrete models, [23]. Clearly one important and fundamental ingredient in such a programme is the provision of as much information on the quantum theory as is calculationaly possible. Various tools exist to achieve this, one of which is the aforementioned explicit perturbation theory coupled with ϵ -expansion techniques. A second is the large N analysis, where N is the number of fundamental fields of the theory, in which one directly calculates d -dimensional expressions for the critical exponents order by order in $1/N$. Indeed there has been intense activity in this area in the last few years, [11-22], with the most successful being the application of Vasil’ev’s self consistency technique, [24, 25], to four fermi theories, [11], allowing the calculation of critical exponents both to $O(1/N^2)$ and $O(1/N^3)$, [11,12,16-19]. Moreover the beauty of the latter technique is that since it provides results in d -dimensions it contributes information to the various problems mentioned earlier *simultaneously*.

In this letter we *complete* the application of the *full* conformal bootstrap programme to four fermi type theories, [17, 18], by deriving the $O(1/N^3)$ expression for the anomalous dimension of the fermion in the Nambu–Jona-Lasinio model with $SU(2) \times SU(2)$ continuous chiral symmetry. Such a calculation, in arbitrary dimensions, is necessary for proving the equivalence of that model with the Gell-Mann–Lévy σ model, [26], which also possesses an $SU(2)$ chiral symmetry. The latter has been used as an effective theory to describe nucleons. The lagrangian of the theory we consider is, [6],

$$L = i\bar{\psi}^{iI}\not{\partial}\psi^{iI} + \sigma\bar{\psi}^{iI}\psi^{iI} + i\pi^a\bar{\psi}^{iI}\lambda_{IJ}^a\gamma^5\psi^{iJ} - \frac{1}{2g^2}(\sigma^2 + \pi^{a2}) \quad (1)$$

where ψ^{iI} is the fermion field with $1 \leq i \leq N$, $1 \leq I \leq M$, $1 \leq a \leq (M^2 - 1)$, λ_{IJ}^a are the generators of, for the moment $SU(M)$, g is the coupling constant and σ and π^a are auxiliary bosonic fields. The 3-point vertex form is used here as it is more appropriate for applying the conformal bootstrap. We note our conventions are $\text{Tr}(\lambda^a\lambda^b) = 4T(R)\delta^{ab}$, $\lambda^a\lambda^a = 4C_2(R)I$ and $f^{acd}f^{bcd} = C_2(G)\delta^{ab}$ with $T(R) = \frac{1}{2}$, $C_2(R) = (M^2 - 1)/2M$ and $C_2(G) = M$ for the group $SU(M)$. Although we have noted the lagrangian for the more general case $SU(M) \times SU(M)$ we will only consider $M = 2$ in detail here. The reason for this is that the models with $M = 2$ and $M > 2$ have distinct properties which only became evident in recent $O(1/N^2)$ calculations, [27]. For example, the anomalous dimensions of the σ and π^a fields are only equal in the abelian case, $U(1) \times U(1)$, and for $M = 2$. For $M > 2$, the equality is not present. Mathematically this division can be traced to the totally symmetric tensor d_{abc} which is zero for $SU(2)$ but not for $SU(M)$, $M > 2$. Further, in the calculation of the β -function exponent at $O(1/N^2)$, [27], one cannot solve the self consistency equation to deduce a simple expression for $M > 2$ as compared to $M = 2$ where a closed analytic solution emerges naturally. At $O(1/N)$ the self consistency formalism which was used, quite correctly reproduced results in agreement with [14]. Therefore we will concentrate here on the calculation of the fermion anomalous dimension for the model $M = 2$ and comment on the situation with $M > 2$ later.

As a preliminary we give our notation and first recall that the conformal propagators of the fields of (1), which are the starting point of the bootstrap method, are, in the asymptotic region of coordinate space $x \rightarrow 0$, [19],

$$\psi(x) \sim \frac{A\not{x}}{(x^2)^\alpha} \quad , \quad \sigma(x) \sim \frac{B}{(x^2)^\beta} \quad , \quad \pi(x) \sim \frac{C}{(x^2)^\gamma} \quad (2)$$

Here A , B and C are the amplitudes of the fields and the critical indices are defined, through dimensional analysis, to be

$$\alpha = \mu + \frac{1}{2}\eta, \quad \beta = 1 - \eta - 2\Delta_\sigma, \quad \gamma = 1 - \eta - 2\Delta_\pi, \quad (3)$$

where $d = 2\mu$, η is the fermion anomalous dimension which we calculate to $O(1/N^3)$ here and Δ_σ and Δ_π are the 3-vertex anomalous dimensions which satisfy, [19],

$$2\alpha + \beta = 2\mu + 1 - 2\Delta_\sigma, \quad 2\alpha + \gamma = 2\mu + 1 - 2\Delta_\pi \quad (4)$$

As already noted $\Delta_\sigma = \Delta_\pi$ to $O(1/N^2)$ and in particular, with $\Delta \equiv \Delta_\sigma = \Delta_\pi$, and, for example, $\eta = \sum_{i=1}^{\infty} \eta_i/N^i$, [14, 21, 27],

$$\begin{aligned} \eta_1 &= -\frac{\Gamma(2\mu-1)}{\Gamma(\mu+1)\Gamma(\mu)\Gamma(1-\mu)\Gamma(\mu-1)} \\ \Delta_1 &= -\frac{\mu\eta_1}{4(\mu-1)} \\ \eta_2 &= \eta_1^2 \left[\frac{(\mu-2)\Psi}{2(\mu-1)} + \frac{1}{2\mu} + \frac{2}{(\mu-1)} - \frac{3\mu}{4(\mu-1)^2} \right] \\ \Delta_2 &= -\frac{\mu\eta_1^2}{16(\mu-1)^2} \left[3\mu(\mu-1)\Theta + 2(\mu-2)\Psi \right. \\ &\quad \left. + \frac{(5\mu-1)(2\mu^2-5\mu+4)}{(\mu-1)} \right] \end{aligned} \quad (5)$$

where $\Psi(\mu) = \psi(2\mu-1) - \psi(1) + \psi(2-\mu) - \psi(\mu)$, $\Theta(\mu) = \psi'(\mu) - \psi'(1)$ and $\psi(x)$ is the logarithmic derivative of the Γ -function.

From (2) the asymptotic scaling forms of the 2-point function, which are also required, are

$$\begin{aligned} \psi^{-1}(x) &\sim \frac{r(\alpha-1)\not{x}}{A(x^2)^{2\mu-\alpha+1}} \\ \sigma^{-1}(x) &\sim \frac{p(\beta)}{B(x^2)^{2\mu-\beta}} \\ \pi^{-1}(x) &\sim \frac{p(\gamma)}{B(x^2)^{2\mu-\gamma}} \end{aligned} \quad (6)$$

where $p(\alpha) = a(\alpha-\mu)/[\pi^{2\mu}a(\beta)]$, $q(\alpha) = \alpha p(\alpha)/(\mu-\alpha)$ and $a(\alpha) = \Gamma(\mu-\alpha)/\Gamma(\alpha)$. These results, (5), have been deduced using the self consistency

approach of [24, 12] where one solves the skeleton Schwinger Dyson equations with dressed propagators but undressed vertices. For the $O(N)$ Gross Neveu model these results have been reproduced *exactly* using the *full* conformal bootstrap programme which will be used here, [28,29,30,19,16]. The difference in this latter approach is that one solves instead the 3-point vertex function whose equivalent Dyson representation is in terms of graphs with both dressed propagators and now dressed vertices. The latter feature reduces substantially the number of Feynman diagrams needed to be computed even at $O(1/N^3)$. The general structure for the present case is illustrated in fig. 1 where the wavy line represents either the σ or π^a fields. The original bootstrap equations for a ϕ^3 style theory have been derived in [28, 30] and extended to the fermion case in [17, 19]. Rather than derive them explicitly for (1) we simply state them as there are no major obstacles in extending [18] for (1).

We denote by V_σ and V_π the values of the respective vertex functions and each will be a function of the exponents α , β and γ as well as various combinations of the amplitudes $z = f_\sigma A^2 B$ and $y = f_\pi A^2 C$. Here f_σ and f_π denote the amplitudes of the respective Polyakov conformal triangle, [29], whose origin is as follows. From (4) the sum of the exponents of the lines meeting at a vertex are $2\mu + 1 - \Delta$, where $\Delta = O(1/N)$. Therefore, recalling the dimensionality of the integration measure associated with a vertex, the overall dimension of any vertex is $-\Delta$ which is non-zero. Consequently one cannot apply directly the conformal integration technique known as uniqueness, [31], or conformal transformations on the integral representation of the graphs in the expansion contained in fig. 1. To circumvent this difficulty in the conformal bootstrap solution, one replaces each vertex by a Polyakov conformal triangle, [28-30], which is illustrated in fig. 2. The exponents \tilde{a} and \tilde{b} of the internal lines comprising this triangle are chosen in such a way that each internal vertex is unique or conformal. This will therefore allow the use of the aforementioned conformal techniques to calculate each Feynman diagram. As a consequence of representing each vertex by such a triangle, carrying out an integration leads to the observation that the result is proportional to $1/\Delta$ which is a reflection of the deviation from uniqueness.

A further set of variables upon which V_σ and V_π depend are the infinitesimal regularizing parameters ϵ , ϵ' and δ . These are required in the formal derivation of the bootstrap equations, [29], to avoid intermediate infinities and are introduced by setting $\alpha \rightarrow \alpha + 2\delta$, $\beta \rightarrow \beta + 2\epsilon$ and $\gamma \rightarrow \gamma + 2\epsilon'$. (In the situation where the regulators are non-zero, it is still possible to choose the internal propagators of a Polyakov conformal triangle to main-

tain uniqueness at each vertex.) Therefore each of the vertex functions have the formal dependence $V_{\sigma,\pi} = V_{\sigma,\pi}(z, y, \alpha, \beta, \gamma; \delta, \epsilon, \epsilon')$.

Equipped with these vertex functions and (2) and (6), we now write down the formal conformal bootstrap equations for (1) which will be solved to obtain η_3 . First,

$$\begin{aligned} V_\sigma(z, y, \alpha, \beta, \gamma; 0, 0, 0) &= 1 \\ V_\pi(z, y, \alpha, \beta, \gamma; 0, 0, 0) &= 1 \end{aligned} \quad (7)$$

which reflect the fact that the sum of the graphs on the right side of fig. 1 is unity. In practice (7) is used to fix the normalization and give z and y at successive orders, though we note $z_1 = y_1$. Secondly, for general M ,

$$r(\alpha - 1) = zt \left. \frac{\partial V_\sigma}{\partial \delta} \right| + 4yuC_2(R) \left. \frac{\partial V_\pi}{\partial \delta} \right| \quad (8)$$

$$p(\beta) = 2NMzt \left. \frac{\partial V_\sigma}{\partial \epsilon} \right| \quad (9)$$

$$p(\gamma) = 8NT(R)ut \left. \frac{\partial V_\pi}{\partial \epsilon'} \right| \quad (10)$$

where $|$ denotes setting all regulators to zero and

$$\begin{aligned} t &= \frac{\pi^{4\mu} a^2(\alpha - 1) a(\tilde{b}) a(\beta)}{\Gamma(\mu)(\alpha - 1)^2 (\tilde{a} - 1)^2 a(\beta - \tilde{b})} \\ u &= \frac{\pi^{4\mu} a^2(\alpha - 1) a(\tilde{c}) a(\gamma)}{\Gamma(\mu)(\alpha - 1)^2 (\tilde{a} - 1)^2 a(\gamma - \tilde{c})} \end{aligned} \quad (11)$$

where \tilde{a} , \tilde{b} and \tilde{c} are the exponents of the internal lines of the respective conformal triangles. Eliminating t and u gives

$$\begin{aligned} r(\alpha - 1) &= \frac{p(\beta)}{2NM} \left. \frac{\partial V_\sigma}{\partial \delta} \right| \Bigg/ \left. \frac{\partial V_\sigma}{\partial \epsilon} \right| \\ &+ \frac{C_2(R)p(\gamma)}{2NT(R)} \left. \frac{\partial V_\pi}{\partial \delta} \right| \Bigg/ \left. \frac{\partial V_\pi}{\partial \epsilon'} \right| \end{aligned} \quad (12)$$

Thus knowledge of the values of the vertex functions to $O(1/N^2)$ means one can deduce η_3 from (12) as it occurs at the same order in the left side of (12). One simplification occurs in the calculation of the derivatives with respect to the regulators. As noted earlier each conformal triangle yields

a pole in the deviation from uniqueness of the vertex it represents. Therefore an n -vertex graph contributing to V_σ , for example, has the structure $h(\Delta, \epsilon, \epsilon', \delta)/\Delta^{n-2}(\Delta - \delta)(\delta - \epsilon)$ where h is a non-singular function. Thus,

$$\left. \frac{\partial V_\sigma}{\partial \delta} \right| \bigg/ \left. \frac{\partial V_\sigma}{\partial \epsilon} \right| = \left[1 + \Delta \left. \frac{\partial V_\sigma}{\partial \delta} \right|_{\text{res}} \right] \left[1 + \Delta \left. \frac{\partial V_\sigma}{\partial \epsilon} \right|_{\text{res}} \right]^{-1} \quad (13)$$

where res denotes the contribution from differentiating the residue function h of the regularized vertex function. This is important since it is not possible to evaluate exactly all the Feynman diagrams at $O(1/N^2)$ but only the difference defined as,

$$\tilde{\Delta}V \equiv \left. \frac{\partial V}{\partial \delta} \right| - \left. \frac{\partial V}{\partial \epsilon} \right| \quad (14)$$

To complete the calculation the explicit values for $\tilde{\Delta}V_\sigma$ and $\tilde{\Delta}V_\pi$ are required. The values of the basic topologies have been calculated in [16] and it is a straightforward exercise to include the effects of the $SU(M) \times SU(M)$ symmetry. For completeness we record that the values for both V_σ and V_π are the same for $M = 2$ but differ for $M > 2$. For the former case which we are solving here, we record

$$\tilde{\Delta}\Gamma_2 = \frac{\mu\eta_1}{2(\mu-1)^2} \quad (15)$$

$$\begin{aligned} \tilde{\Delta}\Gamma_3 = \frac{5\mu\eta_1}{4(\mu-1)^2} & \left[(\mu-1)(2\mu-1) - (2\mu^2 - 5\mu + 4)\Psi \right. \\ & \left. - \frac{5(2\mu^2 - 5\mu + 4)}{2(\mu-1)} \right] \end{aligned} \quad (16)$$

$$\tilde{\Delta}\Gamma_4 = \frac{\mu\eta_1}{8} \left[3\Theta\Xi + \frac{3\Xi}{(\mu-1)^2} + \frac{2\mu\Psi}{(\mu-1)^2} + \frac{(\mu-8)\Theta}{(\mu-1)} + \frac{2(2\mu-3)}{(\mu-1)^3} \right] \quad (17)$$

$$\begin{aligned} \tilde{\Delta}\Gamma_5 = \frac{\mu\eta_1}{8} & \left[3\Theta\Xi + \frac{3\Xi}{(\mu-1)^2} + \frac{2\mu\Psi}{(\mu-1)^2} - \frac{8\Theta}{(\mu-1)} \right. \\ & \left. - \frac{(2\mu-5)(\mu-2)}{(\mu-1)^3} \right] \end{aligned} \quad (18)$$

where $\Phi(\mu) = \psi'(2\mu-1) - \psi'(2-\mu) - \psi'(\mu) + \psi'(1)$ and $\Xi(\mu) = I(\mu) + 2/3(\mu-1)$ and $I(\mu)$ is related to a 2-loop integral which cannot be given in a closed form in terms of ψ -functions, [25].

Consequently after a little algebra we obtain the arbitrary dimensional expression

$$\frac{\eta_3}{\eta_1^3} = \left[\frac{(\mu-2)^2\Psi^2}{2(\mu-1)^2} - \frac{(\mu-2)^2\Phi}{8(\mu-1)^2} - \frac{3\mu^2\Theta\Psi}{8(\mu-1)} - \frac{3\mu^2\Xi}{16(\mu-1)} \left(\Theta + \frac{1}{(\mu-1)^2} \right) \right]$$

$$\begin{aligned}
& + \frac{\mu\Theta}{4(\mu-1)} \left(\frac{1}{\mu} - \frac{3\mu}{2(\mu-1)} - \frac{(5\mu-4)}{4(\mu-1)} - \frac{1}{2} - \frac{\mu(\mu-16)}{8(\mu-1)} \right) \\
& + \left(\frac{3}{2\mu} - \frac{5\mu}{8} - \frac{3}{16} + \frac{33}{16(\mu-1)} - \frac{71}{16(\mu-1)^2} + \frac{13}{16(\mu-1)^3} \right) \Psi \\
& + \frac{1}{2\mu^2} - \frac{3}{\mu} - \frac{19}{16} - \frac{5\mu}{8} + \frac{3}{2(\mu-1)} \\
& + \left[\frac{19}{8(\mu-1)^2} - \frac{33}{8(\mu-1)^3} + \frac{19}{16(\mu-1)^4} \right]
\end{aligned} \tag{19}$$

in addition to reproducing the results of (5). Explicitly in three dimensions we deduce

$$\eta_3 = \frac{32}{27\pi^6} \left[\frac{189}{2} \zeta(3) - 9\pi^2 \ln 2 - \frac{51}{4} \pi^2 + \frac{1157}{9} \right] \tag{20}$$

where $\zeta(z)$ is the Riemann zeta function.

To conclude, we have now completed the full conformal bootstrap analysis for several four fermi theories as far as is calculationaly possible, ie $O(1/N^3)$. It is worth noting for completeness, however, the situation with other cases. In the model with continuous $U(1) \times U(1)$ chiral symmetry, using the full conformal bootstrap approach discussed here, an expression for η_3 cannot be obtained. In this instance the bootstrap equations themselves become singular, which can be seen in several ways, but stems from the fact that this model contains more symmetries than the case with $M = 2$ since one has additionally $\Delta_1 = 0$. Indeed it is this vanishing of Δ at leading order which means that one cannot obtain non-contradictory solutions for z_1 and y_1 from the vertex normalization equations, (7), [16]. As these are important variables for pushing the bootstrap analysis to $O(1/N^3)$, it appears that one does not initially have a consistent set of equations to solve. Alternatively one can see this phenomenon by expressing the same equations in the general non-abelian case in terms of the group Casimirs and then examining the limit to the abelian model. For the non-abelian model with $M > 2$ we have endeavoured to repeat the above analysis. Like the abelian case it is not possible to deduce a value for η_3 . One complication with the analysis, which prevents the emergence of an expression in arbitrary dimensions, is again related to the presence of the non-zero tensor d_{abc} , similar to the problem which arose in the calculation of the critical β -function slope, [27].

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Figure Captions.

Fig. 1 Expansion of 3-vertex.

Fig. 2 Polyakov conformal triangle.

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